Mathématiques et nombres

Quelques programmes et algorithmes reliés aux mathématiques et aux nombres.

- Théorie des nombres
- Nombre_remarquable
- …

Calculs en précision arbitraire

Les calculs suivants renvoient des nombres avec des décimales bien particulières :

- $1/9^2$ = 0.0123456790123456790123456790123456790123456790123457…
- $1/99^2$ = 0.000102030405060708091011121314151617181920212223242526272829303132333435363738394041424344454647484950515253545556575859606162636465666768697071727374757677787980818283848586878889909192939495969799000102030405060708091011…

Quelle est l'explication de ces particularités. Comment manipuler de tels nombres et les construire ?

L'idée est de s'intéresser au développement en série de Taylor de $1/x^2$ autour de a=1, ou de manière équivalente à la série de Maclaurin de $1/(1-x)^2$

Taylor, pour $f(x) = 1/x^2$:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = 1 - 2(x-1) + 3(x-1)^2 4(x-1)^3 + 5(x-1)^4 + ...$$

Maclaurin :

$$1/(1-x)^2 \approx 1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + 7x^6 + 8x^7 + 9x^8 + ...$$

Ce développement va introduire des décimales particulières si x = 0.1, 0.01 ou 0.001,…

Programme

```
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
"""
'Nombres magiques' 100/81 (et 10000/9801) dont les valeurs montrent des
décimales
consécutives

Librairie de multiprécision : http://mpmath.org/doc/current/basics.html

"""
```

```
from mpmath import mp # multiprecision library
n = 41
mp.dps = n
#mp.pretty = True
print('standard float value : ', 1/0.81)

print('Multiprécision :')
sumt = mp.mpf(0)
for i in range(40, 0, -1):
    t = mp.mpf(i) * (mp.mpf(1)/mp.mpf(10))**mp.mpf(i-1)

    sumt += t
    print(mp.nstr(t, n, strip_zeros=False, min_fixed=-n)[:n],
          mp.nstr(sumt, n, strip_zeros=False, min_fixed=-n)[:n])

print(100/mpf(9**2))

n = 51
mp.dps = n
#mp.pretty = True
print('standard float value : ', 1/0.9801)

print('Multiprécision :')
sumt = mp.mpf(0)
for i in range(25, 0, -1):
    t = mp.mpf(i) * (mp.mpf(1)/mp.mpf(100))**mp.mpf(i-1)

    sumt += t
    print(mp.nstr(t, n, strip_zeros=False, min_fixed=-n)[:n],
          mp.nstr(sumt, n, strip_zeros=False, min_fixed=-n)[:n])

print(10000/mpf(99**2))
```

Résultats

```
standard float value : 1.2345679012345678

Multiprécision :

0.0000000000000000000000000000000000000040
0.0000000000000000000000000000000000000040
0.0000000000000000000000000000000000000390
0.0000000000000000000000000000000000000430
0.0000000000000000000000000000000000003800
0.0000000000000000000000000000000000004230
0.0000000000000000000000000000000000037000
0.0000000000000000000000000000000000041230
0.0000000000000000000000000000000000360000
0.0000000000000000000000000000000000401230
```

0.0000000000000000000000000000000003500000
0.0000000000000000000000000000000003901230
0.0000000000000000000000000000000034000000
0.0000000000000000000000000000000037901230
0.0000000000000000000000000000000330000000
0.0000000000000000000000000000000367901230
0.0000000000000000000000000000003200000000
0.0000000000000000000000000000003567901230
0.0000000000000000000000000000031000000000
0.0000000000000000000000000000034567901230
0.0000000000000000000000000000300000000000
0.0000000000000000000000000000334567901230
0.0000000000000000000000000002900000000000
0.0000000000000000000000000003234567901230
0.0000000000000000000000000028000000000000
0.0000000000000000000000000031234567901230
0.0000000000000000000000000270000000000000
0.0000000000000000000000000301234567901230
0.0000000000000000000000002600000000000000
0.0000000000000000000000002901234567901230
0.0000000000000000000000025000000000000000
0.0000000000000000000000027901234567901230
0.0000000000000000000000240000000000000000
0.0000000000000000000000267901234567901230
0.0000000000000000000002300000000000000000
0.0000000000000000000002567901234567901230
0.0000000000000000000022000000000000000000
0.0000000000000000000024567901234567901230
0.0000000000000000000210000000000000000000
0.0000000000000000000234567901234567901230
0.0000000000000000002000000000000000000000
0.0000000000000000002234567901234567901230
0.0000000000000000019000000000000000000000
0.0000000000000000021234567901234567901230
0.0000000000000000180000000000000000000000
0.0000000000000000201234567901234567901230
0.0000000000000001700000000000000000000000
0.0000000000000001901234567901234567901230
0.0000000000000016000000000000000000000000
0.0000000000000017901234567901234567901230
0.0000000000000150000000000000000000000000
0.0000000000000167901234567901234567901230
0.0000000000001400000000000000000000000000
0.0000000000001567901234567901234567901230
0.0000000000013000000000000000000000000000
0.0000000000014567901234567901234567901230
0.0000000000120000000000000000000000000000
0.0000000000134567901234567901234567901230
0.0000000001100000000000000000000000000000
0.0000000001234567901234567901234567901230
0.0000000010000000000000000000000000000000

0.00000001123456790123456790123456790123 0
0.00000009000000000000000000000000000000 0
0.00000010123456790123456790123456790123 0
0.00000080000000000000000000000000000000 0
0.00000090123456790123456790123456790123 0
0.00000700000000000000000000000000000000 0
0.00000790123456790123456790123456790123 0
0.00006000000000000000000000000000000000 0
0.00006790123456790123456790123456790123 0
0.00050000000000000000000000000000000000 0
0.00056790123456790123456790123456790123 0
0.00400000000000000000000000000000000000 0
0.00456790123456790123456790123456790123 0
0.03000000000000000000000000000000000000 0
0.03456790123456790123456790123456790123 0
0.20000000000000000000000000000000000000 0
0.23456790123456790123456790123456790123 0
1.00000000000000000000000000000000000000 0
1.23456790123456790123456790123456790123 0

1.23456790123456790123456790123456790123 46

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standard float value : 1.0203040506070808

Multiprécision :

0.00000000000000000000000000000000000000000000250
0.00000000000000000000000000000000000000000000250
0.00000000000000000000000000000000000000000024000
0.00000000000000000000000000000000000000000024250
0.00000000000000000000000000000000000000002300000
0.00000000000000000000000000000000000000002324250
0.00000000000000000000000000000000000000000220000000
0.00000000000000000000000000000000000000000222324250
0.00000000000000000000000000000000000000021000000000
0.00000000000000000000000000000000000000021222324250
0.00000000000000000000000000000000000000002000000000000
0.00000000000000000000000000000000000000002021222324250
0.00000000000000000000000000000000000000190000000000000
0.00000000000000000000000000000000000000192021222324250
0.00000000000000000000000000000000000018000000000000000
0.00000000000000000000000000000000000018192021222324250
0.00000000000000000000000000000000000170000000000000000
0.00000000000000000000000000000000000171819202122232425 0
0.00000000000000000000000000000000000160000000000000000
0.00000000000000000000000000000000000161718192021222324250
0.00000000000000000000000000000000000150000000000000000
0.00000000000000000000000000000000000151617181920212223242 50
0.00000000000000000000000000000000000140000000000000000000

```
0.000000000000000000000000141516171819202122232425 0
0.000000000000000000000001300000000000000000000000 0
0.000000000000000000000001314151617181920212223242 50
0.000000000000000000000012000000000000000000000000 0
0.000000000000000000000121314151617181920212223242 50
0.000000000000000000001100000000000000000000000000 0
0.000000000000000000011121314151617181920212223242 50
0.000000000000000000100000000000000000000000000000 0
0.000000000000000001011121314151617181920212223242 50
0.000000000000000090000000000000000000000000000000 0
0.000000000000000910111213141516171819202122232425 0
0.000000000000008000000000000000000000000000000000 0
0.000000000000008091011121314151617181920212223242 50
0.000000000000700000000000000000000000000000000000 0
0.000000000000708091011121314151617181920212223242 50
0.000000000600000000000000000000000000000000000000 0
0.000000000607080910111213141516171819202122232425 0
0.000000005000000000000000000000000000000000000000 0
0.000000005060708091011121314151617181920212223242 50
0.000004000000000000000000000000000000000000000000 0
0.000004050607080910111213141516171819202122232425 0
0.000300000000000000000000000000000000000000000000 0
0.000304050607080910111213141516171819202122232425 0
0.020000000000000000000000000000000000000000000000 0
0.020304050607080910111213141516171819202122232425 0
1.000000000000000000000000000000000000000000000000 0
1.020304050607080910111213141516171819202122232425 0

1.020304050607080910111213141516171819202122232425 26
```

Références

- https://mrob.com/ → 1/99**2
  - https://mrob.com/pub/math/numbers-14.html
  - https://mrob.com/pub/seq/digits.html
- https://fr.wikipedia.org/wiki/S%C3%A9rie_de_Taylor
- https://socratic.org/questions/what-is-the-taylor-series-expansion-of-f-x-1-x-2-at-a-1

- Multiprécision en Python : http://mpmath.org/
  - Documentation

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