## System of linear equations

Numerical methods used to solve such problem allow to introduce and experiment on xTime_complexity, considering cubic time behavior of standard algorithms and i.e. quadratic time solutions using LU decomposition.

## Theory

- खSystem_of_linear_equations
- $\times$ Gaussian_elimination, Gauss and Gauss-Jordan eliminations (diagonalization, triangularization)
- XPivot_element, pivoting
- 区LU_decomposition- 区Triangular_matrix\#Forward_and_back_substitution
- Chapter 2 in the book "Numerical Recipes" :
- 2.0 Introduction
- 2.1 Gauss-Jordan Elimination
- 2.2 Gaussian Elimination with Backsubstitution
- 2.3 LU Decomposition and Its Application
- Python NumPy library : NumPy Reference
- Linear algebra (numpy.linalg) : numpy.linalg.solve
- Time complexity analysis
- Hint : in Python, use the timeit module


## Exercices and applications

- Exercices :
- write a python function for diagonalisation with partial pivoting
- random numbers $\rightarrow$ linear systems
- comparison with numpy standard library
- measurements of execution time to check cubic complexity


## 1D problems with neigbours

- Thermal diffusion and chemical diffusion (transient or stationary) on a regular 1D space with equidistant steps. ODE equations can be writen such a given evolution equation for node \# i only imlies nodes $\mathrm{i}+1$ and $\mathrm{i}-1$
- Using Xtridiagonal Thomas algorithm allows to save computational time thanks to n complexity
- ? Python library with Thomas algorithm


## What you must have learned in this chapter

- Except ill-conditionned, linear systems can be solved "exactly" using linear algebra algorithms in a finite and known number of arithmetic operations.
- The accuracy is determined by the number of numerical figures which are encoded in floating point description
- For a general system of $n$ equations, diagonalisation requires of the order of $n^{3}$ operations. Also for solving a system using these method.
- If the coefficient matrix is the same for different systems (only the independent coefficients are different), it is possible to solve systems with the order of $n^{2}$ operations, if the matrix of coeeficients is decomposed in the product of two triangular matrix (Lower-Upper decomposition). This $\mathrm{n}^{3}$ step is realised only once.


## References:

- Numerical recipes, The Art of Scientific Computing 3rd Edition, William H. Press, Saul A. Teukolsky, William T. Vetterling, Brian P. Flannery, 2007, isbn: 9780521880688
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